Octal-to-Decimal Conversion:

An octal number can be easily converted to its decimal equivalent by multiplying each octal digit by its positional weight. For example:

$372_{(8)}=3\times8^2+7\times8^1+2\times8^0$

$=3\times64+7\times8+2\times1=250_{(10)}$

Example: convert $24.6_{(8)}$ to its decimal equivalent.

$24.6_{(8)}=2\times8^1+4\times8^0+6\times8^{-1}$

$=2\times8+4\times1+6\times0.125=20.75_{(10)}$

Decimal-to-Octal Conversion:

A decimal integer can be converted to octal by using the same repeated division method that have been used in the decimal-to-binary conversion, but with a division factor of 8 instead of 2. An example is shown below:

$\frac{266}{8}$ =33 + Remainder 2

$\frac{33}{8}$ = 4 + Remainder 1

$\frac{4}{8}$ = 0 + Remainder 4

$266_{(10)}= 412_{(8)}$

For decimal fractions, multiplying instead of dividing, writing the carry into the integers position. An example of this is to convert 0.23 into an octal fraction.

0.23×8=1.84 = 0.84 with a carry of 1

0.84×8=6.72 = 0.72 with a carry of 6

0.72×8=5.76 = 0.76 with a carry of 5

The process is terminated after three places; if more accuracy were required, we continue multiplying to obtain more octal digit.

Octal-to-Binary Conversion:

The primary advantage of the octal number system is the ease with which conversion can be made between binary and octal numbers. The conversion from octal to binary is performed by converting each octal digit to its 3-bit binary equivalent. The eight possible digits are converted as indicated in Table (2).

Table (2)

Octal No.	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

For example, $472_{(8)}$ is converted to its binary equivalent as follows:-

$472_{(8)} = 100111010_{(2)}$

4 ↓ **100** 7 ↓ **111** 2 ↓ **010**

Example: convert $34.562_{(8)}$ to its binary equivalent.

$34.562_{(8)} = 011100.101110010_{(2)}$

3 ↓ **011** 4 ↓ **100** . 5 ↓ **101** 6 ↓ **110** 2 ↓ **010**

Binary-to-Octal Conversion:

Converting from binary integers to octal integers is done by grouping the binary bits into groups of three bits starting at the LSB. Then each group is converted to its octal equivalent.

Example: convert $100111010_{(2)}$ to octal system.

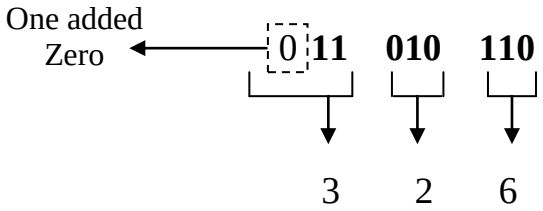
$100111010_{(2)} = 472_{(8)}$

100 ↓ 4 **111** ↓ 7 **010** ↓ 2

Example: convert $11010110_{(2)}$ to its octal equivalent.

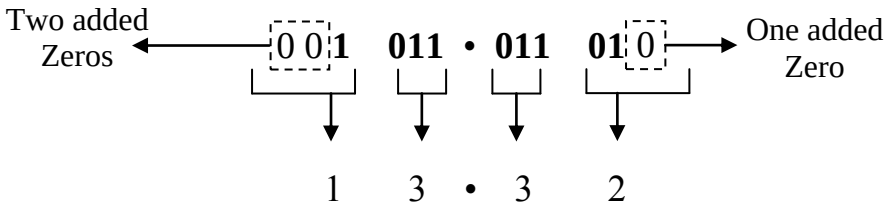
Sometimes the binary number will not have even groups of 3bits. For this case, extra 0's can be added to the left of the MSB of the binary number to fill out the last group as shown below:

$11010110_{(2)} = 326_{(8)}$



Example: convert $1011.01101_{(2)}$ to octal system.

$1011.01101_{(2)} = 13.32_{(8)}$



Hexadecimal Number System:

The hexadecimal system uses base 16. Thus, it has 16 possible digit symbols. It uses the digits 0 through 9 plus the letters (A,B,C,D,E and F) as the 16 digit symbols. Table (3) shows the relationships among hexadecimal, decimal, and binary digits.

Table (3)

Hexadecimal	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110

7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hex-to-Decimal Conversion:

a hex number can be converted to its decimal equivalent by using the fact that each hex digit position has a weight that is a power of 16. the LSD has a weight of $16^0 = 1$, the next higher digit has a weight of $16^1 = 16$, the next higher digit has a weight of $16^2 = 256$, and so on. The conversion is demonstrated in the examples below:

$$356_{(16)} = 3\times16^2 + 5\times16^1 + 6\times16^0$$
$$= 768 + 80 + 6 = 854_{(10)}$$

$$2AF_{(16)} = 2\times16^2 + 10\times16^1 + 15\times16^0$$
$$= 512 + 160 + 15 = 687_{(10)}$$

Decimal-to-Hex Conversion:

Recall that we did decimal-to-binary conversion using repeated division by 2, and decimal-to-octal conversion using repeated division by 8. Likewise, decimal-to-Hex conversion can be done using repeated division by 16.

Example: convert $423_{(10)}$ to hex.

$\frac{423}{16}$ =26 + Remainder 7

$\frac{26}{16}$ = 1 + Remainder 10

$\frac{1}{16}$ = 0 + Remainder 1

$423_{(10)} = 1A7_{(16)}$

Example: convert $214_{(10)}$ to hex.

$\frac{214}{16}$ =13 + Remainder 6

$\frac{13}{16}$ = 0 + Remainder 13

$214_{(10)} = D6_{(16)}$

Hex-to-Binary Conversion:

Each Hex digit is converted to its 4-bit binary equivalent. This is illustrated below for $9F2_{(16)}$.

$9F2_{(16)} = 100111110010_{(2)}$

9 F 2

↓ ↓ ↓

1001 1111 0010

Binary-to-Hex Conversion:

This conversion is just the reverse of the process of Hex-to-Decimal conversion. The binary number is grouped into groups of 4bits, and each group is converted to its equivalent hex digit as in the example below:

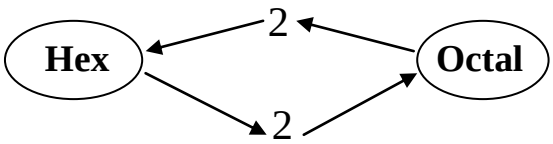
$101110100110_{(2)} = BA6_{(16)}$

1011 1010 0110

↓ ↓ ↓

B A 6

Hex-to-Octal Conversion:



Arithmetic Operation

Addition of Binary Numbers:

The addition of two binary numbers is performed in exactly the same manner as the addition of decimal numbers. Only four cases can occur in adding the two binary digits (bits) in any position. They are:

0+0=0
1+0=1
0+1=1
1+1=0 with carry 1
1+1+1=1 with carry 1

Examples:

011 (3)	1001 (9)	11.011 (3.375)	1010 (10)
+ 110 (6)	+ 1111 (15)	+ 10.110 (2.750)	+ 1101 (13)
1001 (9)	11000 (24)	110.001 (6.125)	10111 (23)

Subtraction of Binary Numbers (Using Direct Method):

The four basic rules for subtracting binary digits are:

0-0=0
1-1=0
1-0=1
0-1=1 with borrow 1

Examples:

when 1 is borrowed, a 0 is left .

when 1 is borrowed, making 10 instead of 0.

11 (3)	11 (3)	101 (5)
- 01 (1)	- 10 (2)	- 011 (3)
10 (2)	01 (1)	010 (2)

Subtraction of Binary Numbers (Using Complement Method):

The 1's complement and the 2's complement of a number are important because they permit the representation of negative numbers. The method of 2's complement arithmetic is used in computer to handle negative numbers

- 1's complement: the 1's complement form of any binary number is obtained simply by changing each 0 in the number to a 1 and each 1 to a 0. In other word, change each bit to its complement. For example:

1 0 1 1 0 1	Binary No.	0 1 1 0 1 0	Binary No.
↓ ↓ ↓ ↓ ↓ ↓		↓ ↓ ↓ ↓ ↓ ↓	
0 1 0 0 1 0	1's complement	1 0 0 1 0 1	1's complement

- 2's complement: the 2's complement form of a binary number is formed simply by taking the 1's complement of the number and adding 1 to the least significant bit position.

2's complement = (1's complement) +1

Example: find 2's complement of 10110010.

1 0 1 1 0 0 1 0	binary number.
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	
0 1 0 0 1 1 0 1	1's complement
+ 1	adding 1
0 1 0 0 1 1 1 0	2's complement

Example: find $11010_{(2)} - 10000_{(2)}$ using 1's complement method (Case 1).

As long as the carry appear, the number is positive and a carry must be added to the result.

```
  11010
+ 01111     1's complement of 10000
 101001
+     1
 01010
```

$11010_{(2)} - 10000_{(2)} = 01010_{(2)}$

Example: find $10000_{(2)} - 11010_{(2)}$ using 1's complement method (Case 2).

As long as no carry appear, the number is negative, then 1's complementing of the final result in needed.

```
  10000
+ 00101    1's complement of 11010
  10101
  ↓↓↓↓↓  1's complement
  01010
```

$$10000_{(2)} - 11010_{(2)} = -\,01010_{(2)}$$

Example: find $11010_{(2)} - 10000_{(2)}$ using 2's complement method (Case 3).

As long as the carry appear, the number is positive and a carry must be discarded

```
  11010
+ 10000    2's complement of 10000
 101010
```

$$11010_{(2)} - 10000_{(2)} = 01010_{(2)}$$

Example: find $10000_{(2)} - 11010_{(2)}$ using 2's complement method (Case 4).

As long as no carry appear, the number is negative, then 2's complementing of the final result in needed.

```
  10000
+ 00110    2's complement of 11010
  10110
  ↓↓↓↓↓  2's complement
  01010
```

$$10000_{(2)} - 11010_{(2)} = -\,01010_{(2)}$$

Multiplication of Binary Numbers:

The numbers in a multiplication are the ***multiplican***d, the ***multiplier***, and the ***product***. These are illustrated in the following decimal multiplication:-

the multiplication rules for binary numbers are:

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$

$$\begin{array}{rl} 8 & \longrightarrow \textit{multiplicand} \\ \times\ 3 & \longrightarrow \textit{multiplier} \\ \hline 24 & \longrightarrow \textit{product} \end{array}$$

Example: find the product of $100_{(2)}$ and $010_{(2)}$.

$$100_{(2)} \times 010_{(2)} = 01000_{(2)}$$

$$\begin{array}{rl} 100 & (4) \\ \times\ 010 & (2) \\ \hline 000 & \\ 1000 & + \\ 00000 & + \\ \hline 01000 & (8) \end{array}$$

Division of Binary Numbers:

The numbers in a division are the ***dividend***, the ***divisor***, and the ***quotient***. These are illustrated in the following standard.

to illustrate, consider the following division examples:

$$\frac{dividend}{divisor} = quotient$$

$$\begin{array}{r} 110 \\ \hline 100 \overline{)\ 11000} \\ 100 \\ \hline 0100 \\ 100 \\ \hline 0000 \end{array}$$

$$\begin{array}{r} 0010.1 \\ \hline 100 \overline{)\ 1010} \\ 100 \\ \hline 00100 \\ 100 \\ \hline 000 \end{array}$$

$$\begin{array}{r} 010.1 \\ \hline 10 \overline{)\ 101} \\ 10 \\ \hline 0010 \\ 10 \\ \hline 00 \end{array}$$